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Pole prescription of higher order induced vertices in Lipatov's QCD effective action

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Abstract

We investigate Lipatov's QCD effective action for the QCD high energy limit and propose a pole prescription for higher order induced vertices. The latter can be used in the evaluation of loop corrections to high energy factorized matrix elements within the effective action approach. The proposed prescription respects the symmetry properties of the unregulated vertices. Explicit results are presented up to third order in the gauge coupling, while an iterative procedure for higher orders is proposed.

1 Introduction

The description of the high energy limit of perturbative QCD is generally given in terms of high energy factorization and the BFKL evolution. The latter resums large logarithms in the center of mass energy $\sqrt{s}$ at leading (LL) [1] and next-to-leading logarithmic (NLL) accuracy [2]. Its derivation is closely related to the observation that QCD scattering amplitudes reveal in the high energy limit an effective t-channel degree of freedom, the reggeized gluon, which couples to the external scattering particles through effective vertices. Determination of both effective couplings and higher order corrections to reggeized gluon exchange is a highly non-trivial task. This is especially true if one attempts to go beyond leading order accuracy – something which is needed for *e.g.* a successful phenomenology of the QCD high energy limit.

An efficient tool to address these questions is given by Lipatov's high energy effective action [3]. It is based on the QCD action with the addition of an induced term. The latter is written in terms of gauge-invariant currents which generate a non-trivial coupling of the gluon to reggeized gluon fields. The effective action allows to address both unitarization of BFKL evolution and the systematic determination of higher order perturbative corrections to high energy QCD amplitudes. While the high energy limit

of QCD tree-level amplitudes can be obtained in a straightforward manner from the effective action [4], in loop corrections a new type of longitudinal divergences, not present in conventional QCD amplitudes, appear. The treatment of these divergences has been at first addressed for leading order transition kernels [5, 6] and the study of the leading order reggeized gluon - gluon - 2 reggeized gluon (RG2R) production vertex in [7,8]. Recently the effective action has been used for the first time for the calculation of NLO corrections to the forward quark jet impact factor [9] and the quark part of the 2-loop gluon trajectory [10], finding precise agreement with previous results in the literature.

In these studies of loop corrections it has been further necessary to give a prescription for circumventing light-cone singularities on the complex plane. The latter appear due to a non-local operator in the induced term of the effective action which describes the coupling of reggeized gluons to conventional gluons. Locality of the effective action in rapidity space and high energy kinematics prevent in principle the light-cone momenta in these denominators to take non-zero values. A prescription for these poles seems not to be necessary. That this is true is evident in the case of tree-level amplitudes in the (Quasi-)Multi-Regge-Kinematics. It seems furthermore reasonable to expect that a similar statement holds for corresponding virtual corrections. On the other hand all regularizations used up till now need a prescription for these denominators. Other regularizations which contain more physical insight are surely possible, but remain to be explored. In this work we therefore give a pole prescription which can be applied in combination with regularization methods used so far.

To leading order (LO) in the gauge coupling g, the order g induced vertex has so far been interpreted as a Cauchy principal value [5–11]. Higher order induced vertices seem to require a more elaborate prescription. In particular interpreting näively every single pole of the higher vertices as a Cauchy principal value destroys parts of the symmetry of the unregulated vertices and can lead to incorrect results, see for instance [5].

In the following we provide a pole prescription of higher order induced vertices which respects the symmetry properties of the unregulated vertices and which can be motivated through the high energy expansion of QCD scattering amplitudes. We will present explicit results up to the order g^3 induced vertex which can be directly used for calculations within the effective action such as the determination of the gluon part of the 2-loop gluon Regge trajectory, corrections to gluon induced production processes and studies of multiple reggeized gluon exchanges within the effective action.

The paper is organized as follows: in sec. 2 we give a short introduction to Lipatov's effective action, including a summary of unregulated Feynman rules. In sec. 3 we present a derivation of the proposed pole prescription and provide explicit expressions up to third order in the gauge coupling. Finally in sec. 4 conclusions and suggestions for future work are presented. The appendix contains a simple QCD example which illustrates the relation of the chosen pole prescription with underlying QCD amplitudes.

2 The effective action of high energy QCD

To set the notation used in the following for the effective action, it is useful to have in mind a partonic elastic scattering process $p_a + p_b \to p_1 + p_2$ with $p_a^2 = 0 = p_b^2$ light-like momenta and $s = (p_a + p_b)^2 = 2p_a \cdot p_b$ the squared center of mass energy. We then define light-like four vectors $n^\pm$ with $n^+ \cdot n^- = 2$, related to the momenta of scattering partons by the re-scaling, $n^+ = 2p_b/\sqrt{s}$ and $n^- = 2p_a/\sqrt{s}$. The Sudakov decomposition of a general four vector k^μ reads $k = k^+ n^-/2 + k^- n^+/2 + \boldsymbol{k}$, where $k^\pm = n^\pm \cdot k$ and $\boldsymbol{k}$ is transverse w.r.t. the initial scattering axis. The effective action is given as the sum of two terms, $S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind.}}$, the QCD action and the induced term. The latter describes the coupling of the reggeized gluon field $A_\pm(x) = -it^a A_\pm^a(x)$ to the gluonic field $v_\mu(x) = -it^a v_\mu^a(x)$. It reads

$$S_{\text{ind.}}[v_\mu, A_\pm] = \int \mathrm{d}^4x \,\mathrm{tr}\left[\Big(W_+[v(x)] - A_+(x)\Big)\partial_\perp^2 A_-(x)\right] \\ + \int \mathrm{d}^4x \,\mathrm{tr}\left[\Big(W_-[v(x)] - A_-(x)\Big)\partial_\perp^2 A_+(x)\right]. \tag{1}$$

The infinite number of couplings of the gluon field to the reggeized gluon field are contained in two functionals $W_\pm[v]$, which are defined through the following operator definition

$$W_\pm[v] = v_\pm \frac{1}{D_\pm}\partial_\pm \qquad \text{where} \qquad D_\pm = \partial_\pm + g v_\pm. \tag{2}$$

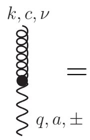
For perturbative calculations, the following expansion in the gauge coupling g holds,

$$W_\pm[v] = v_\pm - g v_\pm \frac{1}{\partial_\pm} v_\pm + g^2 v_\pm \frac{1}{\partial_\pm} v_\pm \frac{1}{\partial_\pm} v_\pm - \ldots \tag{3}$$

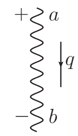
The determination of a suitable regularization of the above operators $1/\partial_\pm$ at zero is then the main goal of this work. Note that the reggeized gluon fields are special in the sense that they are invariant under local gauge transformations, while they transform globally in the adjoint representation of the $\mathrm{SU}(N_c)$ gauge group. In addition strong ordering of longitudinal momenta in high energy factorized amplitudes leads to the following kinematical constraint of the reggeized gluon fields,

$$\partial_+ A_-(x) = \partial_- A_+(x) = 0, \tag{4}$$

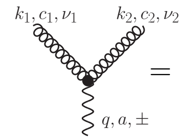
which is always implied. Feynman rules of the high energy effective action have been determined in [4]. We depict them in the following using curly lines for the conventional QCD gluon field and wavy (photon-like) lines for the reggeized gluon field. Following the convention of [4], the Feynman rules of the effective action are given by the conventional QCD Feynman rules and an infinity number of induced vertices, which include



$$= -i\boldsymbol{q}^2 \delta^{ac} (n^{\pm})^{\nu}, \qquad k^{\pm} = 0.$$



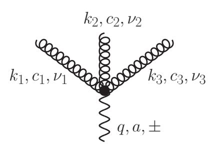
$$= \delta^{ab} \frac{i/2}{\boldsymbol{q}^2}$$



$$= g f^{c_1 c_2 a} \frac{\boldsymbol{q}^2}{k_1^{\pm}} (n^{\pm})^{\nu_1} (n^{\pm})^{\nu_2}, \qquad k_1^{\pm} + k_2^{\pm} = 0$$

(a) (b) (c)

Figure 1: The direct transition vertex (a), the reggeized gluon propagator (b) and the unregulated order g induced vertex (c)



$$= ig^2 \boldsymbol{q}^2 \left(\frac{f^{a_3 a_2 e} f^{a_1 e a}}{k_3^{\pm} k_1^{\pm}} + \frac{f^{a_3 a_1 e} f^{a_2 e a}}{k_3^{\pm} k_2^{\pm}} \right) (n^{\pm})^{\nu_1} (n^{\pm})^{\nu_2} (n^{\pm})^{\nu_3}$$

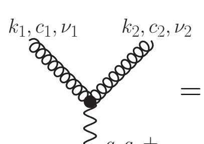
$$k_1^{\pm} + k_2^{\pm} + k_3^{\pm} = 0$$

Figure 2: The unregulated order g^2 induced vertex

a direct transition term between gluon and reggeized gluon, fig. 1.a. Within this direct transition picture, the propagator of the reggeized gluon receives at tree-level a correction due to the projection of the gluon propagator on the reggeized gluon states which we absorb here into the bare reggeized gluon propagator, fig. 1.b. Higher order reggeized gluon - n gluon transition vertices are up to $\mathcal{O}(g^3)$ given by figs. 1.c, 2 and 3. Note that there exists a general iterative formula for the $\mathcal{O}(g^n)$ vertex which is contained in [4]. All of these vertices obey Bose-symmetry, *i.e.* symmetry under simultaneous exchange of color, polarization and momenta of the external gluons of the order g^n vertex. This can be verified making use of the constraint $\sum_i^{n+1} k_i^{\pm} = 0$, which is a direct consequence from eq. (4), and the Jacobi identity.

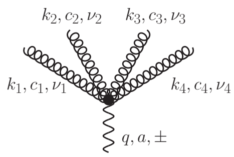
3 Pole prescription for induced vertices

In its original formulation the effective action does not specify a prescription for the poles of induced vertices. A common choice adapted to the induced vertex fig. 1.c is to interpret the pole as a Cauchy principal value, [5, 6, 8, 9]



$$= g f^{c_1 c_2 a} \frac{\boldsymbol{q}^2}{[k_1^{\pm}]} (n^{\pm})^{\nu_1} (n^{\pm})^{\nu_2}, \qquad \frac{1}{[k_1^{\pm}]} \equiv \frac{1}{2} \left(\frac{1}{k_1^{\pm} + i\epsilon} + \frac{1}{k_1^{\pm} - i\epsilon} \right). \tag{5}$$

This choice has the advantage that it maintains the symmetry of the vertex without pole prescription; in particular both Bose-symmetry of the unregulated vertex and



$$
\begin{aligned}
= \quad & g^3 \boldsymbol{q}^2 \Bigg[\frac{f^{c_4 c_3 e_2}}{k_4^\pm} \left(\frac{f^{e_2 c_1 e_1} f^{c_2 e_1 a}}{(k_1^\pm + k_2^\pm) k_2^\pm} + \frac{f^{e_2 c_2 e_1} f^{c_1 e_1 a}}{(k_1^\pm + k_2^\pm) k_1^\pm} \right) \\
& + \frac{f^{c_4 c_1 e_2}}{k_4^\pm} \left(\frac{f^{e_2 a_2 e_1} f^{c_3 e_1 a}}{(k_3^\pm + k_2^\pm) k_3^\pm} + \frac{f^{e_2 a_3 e_1} f^{c_2 e_1 a}}{(k_3^\pm + k_2^\pm) k_2^\pm} \right) + \\
& + \frac{f^{c_4 c_2 e_2}}{k_4^\pm} \left(\frac{f^{e_2 c_1 e_1} f^{c_3 e_1 a}}{(k_3^\pm + k_1^\pm) k_2^\pm} + \frac{f^{e_2 c_2 e_1} f^{c_1 e_1 a}}{(k_3^\pm + k_1^\pm) k_3^\pm} \right) \Bigg] (n^\pm)^{\nu_1} (n^\pm)^{\nu_2} (n^\pm)^{\nu_3} (n^\pm)^{\nu_4}
\end{aligned}
$$

$$
k_1^\pm + k_2^\pm + k_3^\pm + k_4^\pm = 0
$$

Figure 3: The unregulated order g^3 induced vertex

its anti-symmetry under the substitution $k_1^\pm \to -k_1^\pm$ are kept. The latter property is of importance as it can be directly related to negative signature of the reggeized gluon, see [11]. A straightforward extension of the Cauchy principal value prescription to higher order vertices interprets every separate pole as a Cauchy principal value. However, at least if näively applied to the vertices in figs. 2 and 3, such a prescription violates Bose-symmetry and can lead to wrong results, see for instance [5]. This is mainly due to the fact that Cauchy principal values do not obey the eikonal identity in an algebraic sense. One has instead an additional term containing the product of two delta-functions,

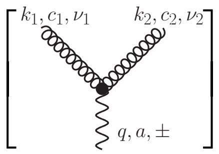
$$
\frac{1}{[k_1^\pm][k_1^\pm + k_2^\pm]} + \frac{1}{[k_1^\pm][k_1^\pm + k_2^\pm]} = \frac{1}{[k_1^\pm][k_2^\pm]} + \pi^2 \delta(k_1^\pm)\delta(k_2^\pm), \tag{6}
$$

see also [12]. In the following section we suggest a prescription which both respects Bose-symmetry and negative signature of the reggeized gluon and therefore maintains the symmetry properties of the unregulated vertices.

3.1 Pole prescription for order g induced vertex

The most straightforward definition is to replace the operator $D_\pm$ in eq. (1) by its regulated counter part $D_\pm - \epsilon$. This is a commonly used choice in other effective theories with Wilson line like operators in the Lagrangian, see for instance [13]. It has the advantage that it can be related to the prescription which arises for these poles from an high energy expansion of QCD diagrams. Up to the order g^2 induced vertex this is explicitly demonstrated in Appendix A, see also the discussion in [14]. For this

regulation, the regulated order g induced vertex reads,



$$
\begin{aligned}
\Big[\ \cdots\ \Big]' &= ig\boldsymbol{q}^2 2\left(\frac{\mathrm{tr}(t^{c_1}t^{c_2}t^a)}{k_2^\pm + i\epsilon} + \frac{\mathrm{tr}(t^{c_2}t^{c_1}t^a)}{k_1^\pm + i\epsilon}\right) n^\pm_{\nu_1} n^\pm_{\nu_2} \\
&= \boldsymbol{q}^2\left[\frac{f^{a_1a_2c}}{2}\left(\frac{1}{k_1^\pm + i\epsilon} + \frac{1}{k_1^\pm - i\epsilon}\right) + \frac{d^{a_1a_2c}}{2}\mathrm{sgn}(\epsilon)2\pi\delta(k_1^\pm)\right] n^\pm_{\nu_1} n^\pm_{\nu_2},
\end{aligned} \tag{7}
$$

with the condition $k_2^\pm = -k_1^\pm$ implied. Eq. (7) coincides with eq. (5) up to the second term in the second line proportional to the symmetric structure constant d^{abc}. The symmetric color structure, the dependence on the sign of ϵ and its symmetry behavior under $k_1^- \to -k_1^-$ indicate that this term corresponds to a reggeized gluon with positive signature. This is in contrast with the original formulation of the effective action which contains only negative signatured reggeized gluons. While it seems at first natural to use eq. (7) instead of eq. (5) and to extend the effective action to reggeized gluons with positive signature, we do not make use of this possibility in the following and discard terms with symmetric color structure. This choice is due to the following reasons:

(i) The dependence on the sign of ϵ leads to a potential conflict with factorization of QCD amplitudes in the high energy limit. It denotes a dependence of the induced vertex on the energy of the high energy factorized scattering parton (for an illustrative QCD example we refer to Appendix A). From the point of view of the effective Lagrangian this dependence on the sign of ϵ can be understood as a violation of hermicity due to the simple replacement $D_\pm \to D_\pm - \epsilon$.

(ii) A further problem is connected with symmetric color tensors such as d^{abc} itself. Unlike color tensors build from anti-symmetric SU(N_c) structure constants, such as the color tensors of the unregulated induced vertices figs. 1.c, 2, 3, the symmetric tensors depend generally on the SU(N_c) representation of the fields in the Lagrangian[1]. In the case of QCD amplitudes this can be backtracked to the SU(N_c) representation of scattering particles, signaling another potential breakdown of high energy factorization[2].

In the following we therefore take the most conservative choice and define the pole prescription of the induced vertices through the replacement $D_\pm \to D_\pm - \epsilon$ together with a subsequent projection on the 'maximal antisymmetric' sub-sector of its color tensors, which is given in terms of anti-symmetric SU(N_c) structure constants alone. The precise meaning of this projection onto the maximal-anti-symmetric sector will be defined in short. Leaving in eq. (7) aside the projector on the color octet $\mathrm{tr}(\cdot t^a)$, where the dot represents any product of SU(N_c) generators, we deal in case of the

[1]We are particular thankful to L. N. Lipatov for drawing our attention to this point.

[2]That there exists indeed a dependence on the representation of scattering particles apart from normalization factors has been directly observed in [15], where an additional contribution for scattering particles in the adjoint representation has been found, which is not present for the fundamental representation.

induced vertex of order g, with a color tensor with two adjoint indices c_1 and c_2. The maximal anti-symmetric sub-sector of order two is then given by the commutator of the two generators, while its symmetric counterpart is defined as the anti-commutator[3]. Projection on the maximal anti-symmetric sub-sector corresponds therefore for the order g induced vertex to dropping the symmetric term proportional to d^{abc} in eq. (7), which leaves us with the commonly used pole prescription eq. (5).

3.2 Generalization to the order g^2 induced vertex

To generalize this projection to higher order induced vertices, we need to find at first an appropriate basis that generalizes the decomposition in commutator and anti-commutator of the previous subsection. This basis requires two elements with a double-commutator, which then yield the color structure of the induced vertex fig. 2. In the following we use a short-cut notation

$$[1, 2] = [t^{c_1}, t^{c_2}], \qquad\qquad S_n(1 \ldots n) = \frac{1}{n!} \sum_{i_1, \ldots i_n} t^{c_{i_1}} \ldots t^{c_{i_n}}, \tag{8}$$

denoting the commutator of two generators with color indices c_1 and c_2 and symmetrization of n generators respectively, where in the second expression the sum is taken over all permutations of the numbers $1, \ldots, n$. In this notation, a possible decomposition of a color tensor with three adjoint indices is given by the following basis

$$[[3, 1], 2], \quad [[3, 2], 1], \quad S_2 ([1, 2]3) \quad S_2 ([1, 3]2) \quad S_2 ([2, 3]1) \quad S_3 (123)\,. \tag{9}$$

As a commutator of two $\mathrm{SU}(N_c)$ generators can be expressed in terms of a single generator, expressions such as $S_2([1, 3]2)$ are well defined *e.g.*

$$S_2 ([1, 3]2) = t^{a_1} t^{a_3} t^{a_2} - t^{a_3} t^{a_1} t^{a_2} + t^{a_2} t^{a_1} t^{a_3} - t^{a_2} t^{a_3} t^{a_1}. \tag{10}$$

The terms in eq. (9) contains two double-anti-symmetric, three mixed-symmetric and one totally symmetric element, while the third double-antisymmetric element $[[1, 2], 3]$ can be expressed in terms of $[[3, 1], 2]$, $[[3, 2], 1]$ by means of the Jacobi-identity. The two double-antisymmetric elements $[[3, 1], 2]$, $[[3, 2], 1]$ define a basis of the maximal anti-symmetric subsector of order two. The above decomposition shares some properties with the usual Young-tableau decomposition, while the definition of anti-symmetrization differs in the present case. Apparently the elements $[[3, 1], 2]$, $[[3, 2], 1]$ define a basis of the maximal anti-symmetric sub-sector of a color tensor with three adjoint indices. To obtain the pole prescription of the induced vertex of order g^2, we start from the effective action with the following expression

$$g^2 v_\pm \frac{1}{\partial \pm -\epsilon} v_\pm \frac{1}{\partial_\pm - \epsilon} v_\pm \partial_\sigma^2 A_\mp, \tag{11}$$

[3]Note that without specifying the symmetric counterpart a projection on the anti-symmetric part is meaningless as the remainder is completely arbitrary and therefore also the resulting pole prescription.

which is obtained through the replacement $D_\pm \to D_\pm - \epsilon$ in eq. (1) and subsequent expansion in g. On the level of Feynman rules, it results into the following unprojected induced vertex of order g^2

$$-ig^2 2\boldsymbol{q}^2 \left(\frac{\mathrm{tr}(t^{c_1}t^{c_2}t^{c_3}t^a)}{(k_2^\pm + k_3^\pm + i\epsilon)(k_3^\pm + i\epsilon)} + \frac{\mathrm{tr}(t^{c_2}t^{c_1}t^{c_3}t^a)}{(k_1^\pm + k_3^\pm + i\epsilon)(k_3^\pm + i\epsilon)} \right.$$
$$+ \frac{\mathrm{tr}(t^{c_1}t^{c_3}t^{c_2}t^a)}{(k_2^\pm + k_3^\pm + i\epsilon)(k_2^\pm + i\epsilon)} + \frac{\mathrm{tr}(t^{c_3}t^{c_1}t^{c_2}t^a)}{(k_1^\pm + k_2^\pm + i\epsilon)(k_2^\pm + i\epsilon)}$$
$$\left. + \frac{\mathrm{tr}(t^{c_3}t^{c_2}t^{c_1}t^a)}{(k_1^\pm + k_2^\pm + i\epsilon)(k_1^\pm + i\epsilon)} + \frac{\mathrm{tr}(t^{c_2}t^{c_3}t^{c_1}t^a)}{(k_1^\pm + k_3^\pm + i\epsilon)(k_1^\pm + i\epsilon)} \right) n^\pm_{\nu_1} n^\pm_{\nu_2} n^\pm_{\nu_3}, \tag{12}$$

with $k_1^\pm + k_2^\pm + k_3^\pm = 0$ implied. Leaving aside for the moment the factor $-ig^2 2\boldsymbol{q}^2 n^\pm_{\nu_1} n^\pm_{\nu_2} n^\pm_{\nu_3}$ and also the projection on the color octet, $\mathrm{tr}(\,\cdot\, t^a)$, eq.(12) reads in the basis eq.(9)

$$-\frac{1}{6}[[3,2],1] \left(\frac{2}{(k_3^\pm - i\epsilon)(k_1^\pm + i\epsilon)} + \frac{2}{(k_3^\pm + i\epsilon)(k_1^\pm - i\epsilon)} + \frac{1}{(k_2^\pm + i\epsilon)(k_3^\pm - i\epsilon)} \right.$$
$$\left. + \frac{1}{(k_2^\pm - i\epsilon)(k_3^\pm + i\epsilon)} + \frac{1}{(k_1^\pm + i\epsilon)(k_2^\pm - i\epsilon)} + \frac{1}{(k_1^\pm - i\epsilon)(k_2^\pm + i\epsilon)} \right)$$
$$-\frac{1}{6}[[3,1],2] \left(\frac{2}{(k_3^\pm - i\epsilon)(k_2^\pm + i\epsilon)} + \frac{2}{(k_3^\pm + i\epsilon)(k_2^\pm - i\epsilon)} + \frac{1}{(k_1^\pm + i\epsilon)(k_3^\pm - i\epsilon)} \right)$$
$$\left. + \frac{1}{(k_1^\pm - i\epsilon)(k_3^\pm + i\epsilon)} + \frac{1}{(k_2^\pm + i\epsilon)(k_1^\pm - i\epsilon)} + \frac{1}{(k_2^\pm - i\epsilon)(k_1^\pm + i\epsilon)} \right)$$
$$-\frac{1}{2}S_2([1,2]3) \left(\frac{1}{(k_3^\pm - i\epsilon)(k_1^\pm + i\epsilon)} - \frac{1}{(k_3^\pm + i\epsilon)(k_1^\pm - i\epsilon)} + \frac{1}{(k_2^\pm - i\epsilon)(k_3^\pm + i\epsilon)} \right.$$
$$\left. - \frac{1}{(k_2^\pm + i\epsilon)(k_3^\pm - i\epsilon)} + \frac{1}{(k_1^\pm + i\epsilon)(k_2^\pm - i\epsilon)} - \frac{1}{(k_1^\pm - i\epsilon)(k_2^\pm + i\epsilon)} \right)$$
$$-\frac{1}{2}S_2([1,3]2) \left(\frac{1}{(k_2^\pm - i\epsilon)(k_1^\pm + i\epsilon)} - \frac{1}{(k_2^\pm + i\epsilon)(k_1^\pm - i\epsilon)} + \frac{1}{(k_3^\pm - i\epsilon)(k_2^\pm + i\epsilon)} \right.$$
$$\left. - \frac{1}{(k_3^\pm + i\epsilon)(k_2^\pm - i\epsilon)} + \frac{1}{(k_1^\pm + i\epsilon)(k_3^\pm - i\epsilon)} - \frac{1}{(k_1^\pm - i\epsilon)(k_3^\pm + i\epsilon)} \right)$$
$$-\frac{1}{2}S_2([2,3]1) \left(\frac{1}{(k_3^\pm - i\epsilon)(k_1^\pm + i\epsilon)} - \frac{1}{(k_3^\pm + i\epsilon)(k_1^\pm - i\epsilon)} + \frac{1}{(k_2^\pm + i\epsilon)(k_3^\pm - i\epsilon)} \right.$$
$$\left. - \frac{1}{(k_2^\pm - i\epsilon)(k_3^\pm + i\epsilon)} + \frac{1}{(k_1^\pm - i\epsilon)(k_2^\pm + i\epsilon)} - \frac{1}{(k_1^\pm + i\epsilon)(k_2^\pm - i\epsilon)} \right)$$
$$-S_3(123) \left(\frac{1}{(k_3^\pm - i\epsilon)(k_1^\pm + i\epsilon)} + \frac{1}{(k_3^\pm + i\epsilon)(k_1^\pm - i\epsilon)} + \frac{1}{(k_2^\pm - i\epsilon)(k_3^\pm + i\epsilon)} \right.$$
$$\left. + \frac{1}{(k_2^\pm + i\epsilon)(k_3^\pm - i\epsilon)} + \frac{1}{(k_1^\pm + i\epsilon)(k_2^\pm - i\epsilon)} + \frac{1}{(k_1^\pm - i\epsilon)(k_2^\pm + i\epsilon)} \right). \tag{13}$$

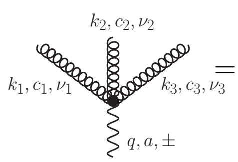
Projection onto the maximal anti-symmetric sub-sector of order three sets then all terms which contain an S_2 or an S_3 symbol to zero, leaving only the color tensors contained in the unregulated vertex fig. 2. The pole structure can be further simplified making use of the eikonal identity

$$\frac{1}{k_1^\pm + i\epsilon}\frac{1}{k_1^\pm + k_2^\pm + i\epsilon} + \frac{1}{k_2^\pm + i\epsilon}\frac{1}{k_1^\pm + k_2^\pm + i\epsilon} = \frac{1}{k_1^\pm + i\epsilon}\frac{1}{k_2^\pm + i\epsilon}, \tag{14}$$

which holds for the above pole prescription not only in the algebraic sense but also in the sense of the theory of distributions [12]. Evaluating commutators and adding the color-octet projection together with the common factor of eq.(12), which corresponds to the substitution

$$\begin{aligned}
[[3,1],2] &\to -ig^2\boldsymbol{q}^2 n^\pm_{\nu_1} n^\pm_{\nu_2} n^\pm_{\nu_3} f^{a_3 a_2 a} f^{a_1 a c}, \\
[[3,2],1] &\to -ig^2\boldsymbol{q}^2 n^\pm_{\nu_1} n^\pm_{\nu_2} n^\pm_{\nu_3} f^{a_3 a_1 a} f^{a_2 a c},
\end{aligned} \tag{15}$$

we obtain for the pole prescription of the order g^2 induced vertex



$$= -ig^2\boldsymbol{q}^2\Big[f^{c_3 c_2 e} f^{c_1 e a} g_2^\pm(3,1,2) + f^{c_3 c_1 e} f^{c_2 e a} g_2^\pm(3,2,1)\Big] n^\pm_{\nu_1} n^\pm_{\nu_2} n^\pm_{\nu_3}, \tag{16}$$

where

$$g_2^\pm(i,j,m) = \left[\frac{-1/3}{k_i^\pm - i\epsilon}\left(\frac{1}{k_m^\pm + i\epsilon} + \frac{1/2}{k_m^\pm - i\epsilon}\right) + \frac{-1/3}{k_i^\pm + i\epsilon}\left(\frac{1}{k_m^\pm - i\epsilon} + \frac{1/2}{k_m^\pm + i\epsilon}\right)\right]. \tag{17}$$

Eq.(17) can be further compactified, making use of the identity,

$$\frac{1}{k^\pm + i\epsilon} - \frac{1}{k^\pm - i\epsilon} = -2\pi i \delta(k^\pm), \tag{18}$$

leading to

$$g_2^\pm(i,j,m) = \left[\frac{-1}{[k_i^\pm][k_m^\pm]} - \frac{\pi^2}{3}\delta(k_i^\pm)\delta(k_m^\pm)\right]. \tag{19}$$

Using the eikonal identity for Cauchy principal values eq. (6) and the condition $k_1^\pm + k_2^\pm + k_3^\pm = 0$ we find that the eikonal function g_2 obeys itself the eikonal identity in the purely algebraic sense *i.e.*

$$g_2^\pm(3,2,1) = -g_2(1,3,2) - g_2(3,1,2). \tag{20}$$

Bose-symmetry of eq. (19) is then easily verified. Invariance of eq. (16) under $\epsilon \to -\epsilon$ and symmetry under $\{k_1^\pm, k_2^\pm, k_3^\pm\} \to \{-k_1^\pm, -k_2^\pm, -k_3^\pm\}$ in analogy to the unregulated vertex fig. 2 are also satisfied by the regulated vertex.

A comment is in order concerning the two delta functions appearing in eq. (19). At first sight they seem to be in conflict with the high energy expansion of underlying QCD amplitudes which requires non-zero $k_i^\pm$, $i = 1, 2, 3$. These delta-functions appear however only due to the identity eq. (18) which allows to reduce eq. (17) into the more compact expression eq. (19). Eq. (17) and the corresponding expressions in eq. (13) are on the other hand free of any delta-functions and can be therefore directly related to an expansion of QCD Feynman diagrams. In addition one should note that any product of two Cauchy principal values can be related to a product of two delta-functions through the identity eq. (6) and therefore demanding complete absence of such terms is hard to achieve.

3.3 Pole prescription of higher order induced vertices

To define the pole prescription of the order g^3 and higher vertices we follow the same pattern. The basis of the color tensor with four adjoint indices $t^{c_1}t^{c_2}t^{c_3}t^{c_4}$ has now 24 independent elements. Again it can be decomposed into a six dimensional maximal anti-symmetric sub-sector which has a basis in terms of combined anti-symmetric structure constants alone and symmetrization of lower dimensional maximal anti-symmetric sub-sectors. A possible decomposition is given by

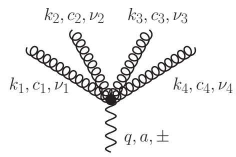
$$
\begin{array}{c|c|c|c|c}
[[[4,1],2],3] & S_2([1,2],[3,4]) & S_2([[1,2],3]4) & S_3([1,2]34) & S_4(1234) \\
[[[4,1],3],2] & S_2([1,3],[2,4]) & S_2([[3,2],1]4) & S_3([1,3]24) & \\
[[[4,2],1],3] & S_2([1,4],[2,3]) & S_2([[3,2],1]4) & S_3([1,4]23) & \\
[[[4,2],3],1] & & S_2([[1,2],4]3) & S_3([2,3]14) & \\
[[[4,3],2],1] & & S_2([[4,2],1]3) & S_3([2,3]14) & \\
[[[4,3],1],2] & & S_2([[1,3],4]2) & S_3([2,4]13) & \\
 & & S_2([[4,3],1]2) & S_3([3,4]12) & \\
 & & S_2([[2,3],4]1) & & \\
 & & S_2([[4,3],2]1) & &
\end{array}
\quad . \tag{21}
$$

To determine the pole prescription of the order g^3 induced vertex we start from the following expression at Lagrangian level

$$
-g^3 v_\pm \frac{1}{\partial_\pm - \epsilon} v_\pm \frac{1}{\partial_\pm - \epsilon} v_\pm \frac{1}{\partial_\pm - \epsilon} v_\pm \partial_\sigma^2 A_\mp, \tag{22}
$$

which is obtained from the replacement $D_\pm \to D_\pm - \epsilon$ and subsequent expansion in g. Re-writing the color tensors of the resulting vertex in terms of the above basis and setting all tensors to zero, apart from the basis elements of the maximal anti-symmetric

sector of order four, contained in the first column of eq. (21), we obtain the following regulated order g^3 induced vertex



$$= -g^3 \boldsymbol{q}^2 n^{\pm}_{\nu_1} n^{\pm}_{\nu_2} n^{\pm}_{\nu_3} n^{\pm}_{\nu_4} \cdot$$

$$\Big[f^{a_4 a_1 d_2} f^{d_2 a_3 d_1} f^{d_1 a_2 c} g_3^{\pm}(4,1,3,2) + f^{a_4 a_1 d_2} f^{d_2 a_2 d_1} f^{d_1 a_3 c} g_3^{\pm}(4,1,2,3)$$
$$+ f^{a_4 a_2 d_2} f^{d_2 a_1 d_1} f^{d_1 a_3 c} g_3^{\pm}(4,2,1,3) + f^{a_4 a_2 d_2} f^{d_2 a_3 d_1} f^{d_1 a_1 c} g_3^{\pm}(4,2,3,1)$$
$$+ f^{a_4 a_3 d_2} f^{d_2 a_1 d_1} f^{d_1 a_2 c} g_3^{\pm}(4,3,1,2) + f^{a_4 a_3 d_2} f^{d_2 a_2 d_1} f^{d_1 a_1 c} g_3^{\pm}(4,3,2,1) \Big], \qquad (23)$$

where the function $g_3^{\pm}(i,j,m,n)$ is defined as

$$g_3^{\pm}(i,j,m,n) = \frac{(-1)}{12} \Bigg\{ \frac{1}{k_i^{\pm} + i\epsilon} \Bigg[\frac{1}{k_n^{\pm} + i\epsilon} \left(\frac{1}{k_n^{\pm} + k_m^{\pm} + i\epsilon} + \frac{1}{k_n^{\pm} + k_m^{\pm} - i\epsilon} \right) +$$
$$\frac{1}{k_n^{\pm} - i\epsilon} \left(\frac{3}{k_n^{\pm} + k_m^{\pm} - i\epsilon} + \frac{1}{k_n^{\pm} + k_m^{\pm} + i\epsilon} \right) \Bigg] + \frac{1}{k_i^{\pm} - i\epsilon} \Bigg[\frac{1}{k_n^{\pm} - i\epsilon} \Bigg(\frac{1}{k_n^{\pm} + k_m^{\pm} - i\epsilon}$$
$$+ \frac{1}{k_n^{\pm} + k_m^{\pm} + i\epsilon} \Bigg) + \frac{1}{k_n^{\pm} + i\epsilon} \left(\frac{3}{k_n^{\pm} + k_m^{\pm} + i\epsilon} + \frac{1}{k_n^{\pm} + k_m^{\pm} - i\epsilon} \right) \Bigg] \Bigg\}. \qquad (24)$$

Similarly to eq. (17) this function g_3 can be written in terms of Cauchy-principal values and delta-functions,

$$g_3^{\pm}(i,j,m,n) = \Bigg(\frac{-1}{[k_i^{\pm}][k_n^{\pm} + k_m^{\pm}][k_n^{\pm}]} - \frac{\pi^2}{3} \delta(k_n^{\pm}) \delta(k_m^{\pm}) \frac{-1}{[k_i^{\pm}]}$$
$$- \frac{\pi^2}{3} \delta(k_n^{\pm}) \delta(k_i^{\pm}) \frac{1}{[k_m^{\pm}]} - \frac{\pi^2}{3} \delta(k_n^{\pm} + k_m^{\pm}) \delta(k_i^{\pm}) \frac{1}{[k_n^{\pm}]} \Bigg). \qquad (25)$$

Using eq.(6), eikonal identities such as

$$g_3^{\pm}(4,1,3,2) = g_3^{\pm}(2,3,1,4) + g_3^{\pm}(2,3,4,1) + g_3^{\pm}(k_2^-, k_1^-, k_3^-, k_4^-), \qquad (26)$$

can be shown to hold, which allow to prove Bose-symmetry of the induced vertex with the above pole-prescription. Invariance under $\epsilon \to -\epsilon$ and anti-symmetry under the substitution $\{k_1^{\pm}, k_2^{\pm}.k_3^{\pm}, k_4^{\pm}\} \to \{-k_1^{\pm}, -k_2^{\pm} - k_3^{\pm}, -k_4^{\pm}\}$ in accordance with the behavior of the unregulated vertex fig. 3 can be shown to hold for the regulated vertex eq. (23). At this stage the general recipe for the construction of the pole prescription of the induced vertices should be clear. The starting point is the projector $P_{\mathrm{A}}^{(1)}$ which acts trivially on the single generator,

$$P_{\mathrm{A}}^{(1)} t^{a_1} = t^{a_1}, \qquad (27)$$

and defines in this way the maximal antisymmetric sector of order one. To arrive at the projector $P_{\rm A}^{(n)}$ which projects the color tensor with n adjoint indices $t^{a_1}....t^{a_n}$ on its maximal anti-symmetric sub-sector we proceed as follows. We first construct a basis for all possible color tensors with n adjoint indices which can be obtained through symmetrization of maximal anti-symmetric sub-sectors of order $n-1$ or lower. The color tensors with n adjoint indices which are orthogonal to this (partial) symmetric sub-space, define then the maximal anti-symmetric sub-sector of order n. $P_{\rm A}^{(n)}$ then projects a generic color tensor with n adjoint indices onto this maximal anti-symmetric sub-sector of order n. We have explicitly verified up to $n = 5$ that this sub-sector has a basis in terms of $(n-1)!$ independent combinations of anti-symmetric SU(N_c) structure constants, as contained in the unregulated induced vertices. Given this definition of projectors, the pole prescription can be given directly at Lagrangian level, if desired. This requires to replace the unregulated operator $W_\pm(v)$ in the induced Lagrangian eq. (1) by

$$W_\pm^\epsilon[v] = \frac{1}{2}\left[\mathcal{P}_{\rm A}\left(v_\pm\frac{1}{D_\pm - \epsilon}\partial_\pm\right) + \mathcal{P}_{\rm A}\left(v_\pm\frac{1}{D_\pm + \epsilon}\partial_\pm\right)\right], \tag{28}$$

where the symmetrization in ϵ has no effect on the resulting Feynman rules and merely serves to ensure hermicity of the regulated Lagrangian. The projector $\mathcal{P}_{\rm A}$ acts order by order in g on the SU(N_c) color structure of the gluonic fields $v_\pm(x) = -it^a v^a(x)$,

$$\begin{aligned}\mathcal{P}_{\rm A}\left(v_\pm\frac{1}{D_\pm - \epsilon}\partial_\pm\right) \equiv -i\bigg(& P_{\rm A}^{(1)}(t^a)v_\pm^a - (-ig)v_\pm^{a_1}\frac{1}{\partial_\pm - \epsilon}v_\pm^{a_2}P_{\rm A}^{(2)}\,(t^{a_1}t^{a_2}) \\ &+ (-ig)^2 v_\pm^{a_1}\frac{1}{\partial_\pm - \epsilon}v_\pm^{a_2}\frac{1}{\partial_\pm - \epsilon}v_\pm^{a_3}P_{\rm A}^{(3)}\,(t^{a_1}t^{a_2}t^{a_3}) - \dots\bigg),\end{aligned} \tag{29}$$

where $P_{\rm A}^{(n)}$ are the projectors of the color tensors with n adjoint indices on the maximal anti-symmetric sub-sector of order n.

4 Summary and conclusion

We derived a pole prescription for the higher order induced vertices of Lipatov's high energy effective action which respects the symmetry properties of the unregulated induced vertices and leads to identical color structure for regulated and unregulated vertices. Explicit expressions have been derived up to the order g^3 induced vertex, while a recipe for the determination of the prescription of the order g^n induced vertices $n \geq 4$ has been proposed. The prescription has the additional advantage that it can be related to an expansion of QCD diagrams in the high energy limit. Future applications of these vertices are numerous: they will be used for the determination of the 2-loop corrections to the gluon Regge trajectory from the effective and the determination of NLO corrections to effective vertices. In addition they are highly relevant to construct

evolution equations and production vertices in the presence of multiple reggeized gluon exchanges which are currently investigated within the context of the effective action.

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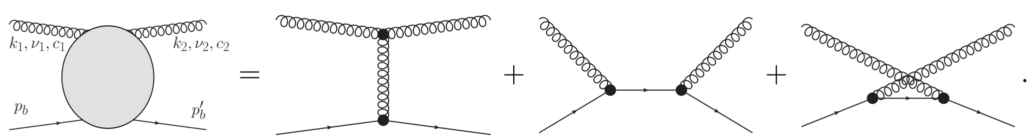
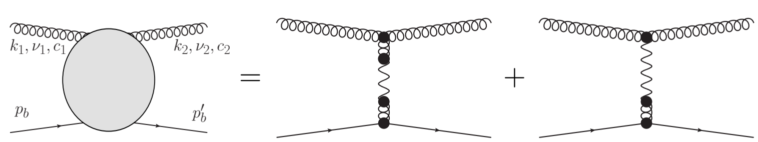
A Pole prescription and high energy limit of QCD

In the following we take a closer look on the relation between the proposed pole prescription and the high energy limit of QCD Feynman diagrams. To start with, we consider the high energy limit of the QCD $gq \to gq$ scattering at tree-level. Within QCD, this process is described by the sum of the following diagrams



(30)

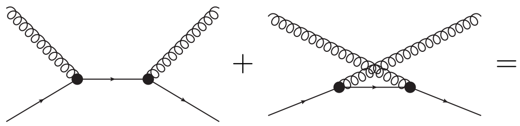
The effective action describes on the other hand the same process (up to corrections suppressed by powers of $s = 2p_a \cdot p_b$) as



$$+\,\mathcal{O}\left(\frac{|t|}{s}\right), \qquad (31)$$

with $t = q^2 = (p_a - p_1)^2$. For covariant gauges, to which we restrict here, the first effective diagram can be identified with the high energy limit of the first QCD diagram, while the second effective diagram which contains the induced vertex of order g can be identified with the high energy limit of the second and third QCD diagram. Leaving

aside the polarization vectors of the gluons one has in the high energy limit



$$
\begin{aligned}
&= \bar{u}(p_b') \left[igt^{c_2}\gamma^{\nu_2} \frac{i(\not{p}_b + \not{k}_1)}{(p_b + k_1)^2 + i\epsilon} igt^{c_1}\gamma^{\nu_1} + igt^{a}\gamma^{\nu_1} \frac{i(\not{p}_b - \not{k}_2)}{(p_b - k_2)^2 + i\epsilon} igt^{c}\gamma^{\nu_2} \right] u(p_b) \\
&= \bar{u}(p_b') ig\not{n}^- \frac{i/2}{\boldsymbol{q}^2} \left[\left(\frac{ig\boldsymbol{q}^2 t^{c_2} t^{c_1}}{k_1^+ + i\epsilon/p_b^-} - \frac{ig\boldsymbol{q}^2 t^{c_1} t^{c_2}}{k_1^+ - i\epsilon/p_b^-} \right) (n^+)^{\nu_1} (n^+)^{\nu_2} \right] u(p_b) + \mathcal{O}\left(\frac{|t|}{s}\right), \quad (32)
\end{aligned}
$$

where the expression in the squared bracket can – up to a projection on the color octet sector – be identified with the order g induced vertex as obtained in eq. (7). This is the core of the statement that the replacement $D_\pm \to D_\pm - \epsilon$ leads to a pole prescription in accordance with the high energy expansion of QCD Feynman diagrams.

While in the case of anti-symmetric color a dependence of the squared bracket on the sign of p_b^- is absent, the same is not true for the symmetric color sector. As a consequence high energy factorization of eq. (32) is only completely realized in the anti-symmetric color sector. The latter changes the sign under $s \to -s$ and therefore corresponds – from the point of view of Regge theory – to a negative signature. The sub-leading symmetric color sector, which is invariant under $s \to -s$ and therefore carries positive signature, keeps a residual dependence on the light-cone energy of the scattering quark and therefore does not factorize completely. In addition the resulting color tensor depends in this case on the representation of the generators t^d in eq. (32). Projecting on the color octet, we obtain for the symmetric sector for generators in the fundamental representation[4] t_F^a the symmetric SU(N_c) structure constant

$$
\mathrm{tr}_F \left(\{t_F^a, t_F^b\} t_F^c \right) = \frac{1}{2} d^{abc}. \tag{33}
$$

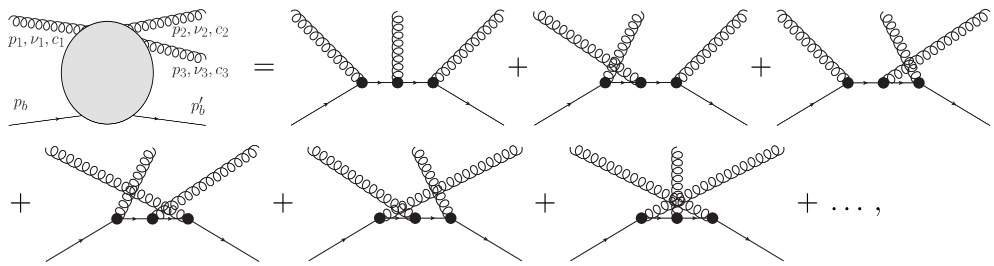
Generators in the adjoint representation t_A^a yield instead a zero result

$$
\mathrm{tr}_A \left(\{t_A^a, t_A^b\} t_A^c \right) = 0. \tag{34}
$$

For the anti-symmetric sector, the commutator leads in both cases to the anti-symmetric structure constant f^{abc}. The above mapping from the prescription $D_\pm \to D_\pm - \epsilon$ to the prescription obtained from Feynman diagrams holds also for higher order induced vertices. For the order g^2 induced vertex one starts in this case with the $gq \to ggq$ scattering process in Quasi-Multi-Regge-Kinematics where the final state gluons are of

[4]We denote in this paragraph generators in the fundamental representation with a label F and generators in the adjoint representation with a label A. Apart from this paragraph t^a always denotes a generator in the fundamental representation.

similar rapidity. The Feynman diagrams which are within Feynman gauge associated with the order g^2 induced vertex read



(35)

where the dots indicates Feynman diagrams which can be associated with combinations of lower induced vertices and pure QCD vertices. Taking now all gluon momenta to be incoming, and leaving again aside the polarization vectors of the gluons one finds for the sum of diagrams depicted diagrams in eq. (35) in the high energy limit

$$
\begin{aligned}
\bar{u}(p_b') ig \not{n}^- u(p_b) \frac{i/2}{\boldsymbol{q}^2} \Bigg[\Bigg(& \frac{-ig^2 \boldsymbol{q}^2 t^{c_3} t^{c_2} t^{c_1}}{(k_1^+ + i\epsilon/p_b^-)(-k_3^+ + i\epsilon/p_b^-)} + \frac{-ig^2 \boldsymbol{q}^2 t^{c_3} t^{c_1} t^{c_2}}{(k_2^+ + i\epsilon/p_b^-)(-k_3^+ + i\epsilon/p_b^-)} \\
& + \frac{-ig^2 \boldsymbol{q}^2 t^{c_2} t^{c_3} t^{c_1}}{(k_1^+ + i\epsilon/p_b^-)(-k_2^+ + i\epsilon/p_b^-)} + \frac{-ig^2 \boldsymbol{q}^2 t^{c_2} t^{c_1} t^{c_3}}{(k_3^+ + i\epsilon/p_b^-)(-k_2^+ + i\epsilon/p_b^-)} \\
& + \frac{-ig^2 \boldsymbol{q}^2 t^{c_1} t^{c_3} t^{c_2}}{(k_2^+ + i\epsilon/p_b^-)(-k_1^+ + i\epsilon/p_b^-)} + \frac{-ig^2 \boldsymbol{q}^2 t^{c_1} t^{c_2} t^{c_3}}{(k_3^+ + i\epsilon/p_b^-)(-k_1^+ + i\epsilon/p_b^-)} \Bigg) \\
& \qquad \cdot (n^+)^{\nu_1} (n^+)^{\nu_2} (n^+)^{\nu_3} \Bigg] + \mathcal{O}\left(\frac{\boldsymbol{q}^2}{s} \right), \qquad (36)
\end{aligned}
$$

from where the relation to expressions such as eq. (12) becomes clear. Similar results hold then in an apparent way for higher order induced vertices.

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